33-rd Spanish Mathematical Olympiad 1997

Second Round
Valencia

First Part

1. Compute the sum of the squares of the first 100 terms of an arithmetic progression, given that their sum is -1 and that the sum of those among them having an even index is 1.

2. A square of side 5 is divided into 25 unit squares. Let A be the set of the 16 interior points of the initial square which are vertices of the unit squares. What is the largest number of points of A no three of which form an isosceles right triangle?

3. For each parabola $y = x^2 + px + q$ intersecting the coordinate axes in three distinct points, consider the circle passing through these points. Prove that all these circles pass through a single point, and find this point.

Second Part

4. Let p be a prime number. Find all integers k for which $\sqrt{k^2 - pk}$ is a positive integer

5. Prove that in every convex quadrilateral of area 1, the sum of the lengths of the sides and diagonals is not smaller than $2(2 + \sqrt{2})$.

6. The exact quantity of gas needed for a car to complete a single loop around a track is distributed among n containers placed along the track. Prove that there exists a position starting at which the car, beginning with an empty tank of gas, can complete a loop around the track without running out of gas. The tank of gas is assumed to be large enough.