26-th Spanish Mathematical Olympiad 1990

Second Round

First Part

1. Prove that $\sqrt{x}+\sqrt{y}+\sqrt{xy}$ is equal to $\sqrt{x}+\sqrt{y+xy+2y\sqrt{x}}$ and compare the numbers
$$\sqrt{3}+\sqrt{10+2\sqrt{3}} \quad \text{and} \quad \sqrt{5+\sqrt{22}}+\sqrt{8-\sqrt{22}+2\sqrt{15-3\sqrt{22}}}.$$

2. Every point of the plane is painted with one of three colors. Can we always find two points a distance 1cm apart which are of the same color?

3. Prove that $\left[\left(4+\sqrt{11}\right)^n\right]$ is odd for every natural number n.

Second Part

4. Prove that the sum
$$\sqrt[3]{\frac{a+1}{2}+\frac{a+3}{6}\sqrt{\frac{4a+3}{3}}}+\sqrt[3]{\frac{a+1}{2}-\frac{a+3}{6}\sqrt{\frac{4a+3}{3}}}$$
is independent of a for $a \geq -\frac{3}{4}$ and evaluate it.

5. On the sides BC, CA and AB of a triangle ABC of area S are taken points A', B', C' respectively such that $AC'/AB = BA'/BC = CB'/CA = p$, where $0 < p < 1$ is variable.
   (a) Find the area of triangle $A'B'C'$ in terms of p.
   (b) Find the value of p which minimizes this area.
   (c) Find the locus of the intersection point P of the lines through A' and C' parallel to AB and AC respectively.

6. There are n points in the plane so that no two pairs are equidistant. Each point is connected to the nearest point by a segment. Show that no point is connected to more than five points.

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