44-th Spanish Mathematical Olympiad 2008

Valencia, March 28-29, 2008

First Part

1. Find two positive integers a and b, when their sum and their least common multiple is given. Find the numbers when the sum is 3972 and the least common multiple is 985928.

2. Let a and b be two real numbers, with $0 < a, b < 1$. Prove that

$$\sqrt{ab^2 + a^2b} + \sqrt{(1-a)(1-b)^2 + (1-a)^2(1-b)} < \sqrt{2}.$$

3. Let $p \leq 3$ be a prime number. Each side of a triangle is divided into p equal parts, and we draw a line from each division point to the opposite vertex. Find the maximum number of regions, every two of them disjoint, that are formed inside the triangle.

Second Part

4. Let p and q be two different prime numbers. Prove that there are two positive integers, a and b, such that the arithmetic mean of the divisors of $n = p^a q^b$ is an integer.

5. Given a circle, two fixed points A and B and a variable point P, all of them on the circle, and a line r, PA and PB intersect r at C and D, respectively. Find two fixed points on r: M and N, such that $CM \cdot DN$ is constant for all P.

6. A plane is painted with seven colors, each point having one color. Is there an inscribed trapezoid whose vertices are all of the same color?

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