20-th Polish Mathematical Olympiad 1968/69

Third Round

First Day

1. Prove that if real numbers a,b,c satisfy the equality
$$\frac{a}{m+2}+\frac{b}{m+1}+\frac{c}{m}=0$$
for some positive number m, then the equation $ax^2+bx+c=0$ has a root between 0 and 1.

2. Given distinct real numbers $a_1,a_2,\dots,a_n$, find the minimum value of the function
$$y=|x-a_1|+|x-a_2|+\cdots+|x-a_n|, \quad x\in\mathbb{R}.$$

3. Show that if natural numbers a,b,p,q,r,s satisfy the conditions
$$qr-ps=1 \quad \text{and} \quad \frac{p}{q}<\frac{a}{b}<\frac{r}{s},$$
then $b\geq q+s$.

Second Day

4. Prove that if a figure has exactly n axes of symmetry in space, then n must be odd.

5. Prove that an octagon, whose all angles are equal and all sides have rational length, has a center of symmetry.

6. For which values of n does there exist a polyhedron having n edges?

www.imomath.com

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com