18-th Polish Mathematical Olympiad 1966/67

Third Round

First Day

1. Real numbers $a_1, a_2, \dots, a_n$ ($n \geq 3$) satisfy the conditions $a_1 = a_n = 0$ and $a_{k-1} + a_{k+1} \geq 2a_k$ for $k = 2, 3, \dots, n-1$. Prove that none of the numbers $a_1, \dots, a_n$ is positive.

2. There are 100 persons in a hall, everyone knowing at least 66 of the others. Prove that there is a case in which among any four some two don't know each other.

3. On the plane are placed two triangles exterior to each other. Show that there always exists a line passing through two vertices of one triangle and separating the third vertex from all vertices of the other triangle.

Second Day

4. There are 100 persons in a hall, everyone knowing at least 67 of the others. Prove that there always exist four of them who know each other.

5. Points A, B, C, D, E in space have the property that
$$AB = BC = CD = DE = EA,$$
$$\angle ABC = \angle BCD = \angle CDE = \angle DEA = \angle EAB.$$
Prove that the points A, B, C, D, E lie on a plane.

6. Prove that if a cyclic polygon with an odd number of sides has all angles equal, then this polygon is regular.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com