17-th Polish Mathematical Olympiad 1965/66

Third Round

First Day

1. Prove that if two cubic polynomials with integer coefficients have an irrational root in common, then they have another common irrational root.
2. Solve in integers the equation $x^4+4y^4=2(z^4+4u^4)$.
3. If nonnegative real numbers $x_1,x_2,\dots,x_n$ satisfy $x_1+\cdots+x_n\le\frac{1}{2}$, prove that
$$(1-x_1)(1-x_2)\cdots(1-x_n)\ge\frac{1}{2}.$$

Second Day

4. Prove that the sum of the squares of the areas of the projections of the faces of a rectangular parallelepiped on a plane is the same for all positions of the plane if and only if the parallelepiped is a cube.
5. Each of the diagonals AD,BE,CF of a convex hexagon $ABCDEF$ bisects the area of the hexagon. Prove that these three diagonals pass through the same point.
6. On the plane are chosen six points. Prove that the ratio of the longest distance between two points to the shortest is at least $\sqrt{3}$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com