14-th Pan-African Mathematical Olympiad

Tunis, Tunisia, 2004

First Day

1. Do there exist positive integers m,n such that $3n^2+3n+7=m^3$?

2. Is $4\sqrt{4-2\sqrt{3}}+\sqrt{97-56\sqrt{3}}$ an integer?

3. One writes 268 numbers around a circle such that the sum of 20 consecutive numbers is always equal to 75. The numbers 3, 4 and 9 are written in positions 17, 83 and 144 respectively. Find the number in position 210.

Second Day

4. Find three real numbers satisfying the following conditions:

   (i) the square of their sum equals the sum of their squares, and

   (ii) the product of the first two numbers equals the square of the third.

5. Each of the digits $1,3,7,9$ occurs at least once in the decimal representation of some positive integer. Prove that one can permute the digits of this integer such that the resulting integer is divisible by 7.

6. A convex quadrilateral $ABCD$ is inscribed in the circle with diameter AB. Let AB and CD meet at I, AD and BC at J, and AC and BD at K, and let N be a point on AB. Prove that $IK \perp JN$ if and only if N is the midpoint of AB.

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