12-th Pan-African Mathematical Olympiad

Pretoria, South Africa, 2002

First Day

1. Find all functions $f : \mathbb{N}_0 \to \mathbb{N}_0$ whose minimum value is 1 and
$$f(f(n)) = f(n) + 1 \quad \text{for all } n \in \mathbb{N}_0.$$

2. Variable points C and D on the respective sides AO and BO of a triangle ABO with a right angle at O satisfy $AC = BD$. Prove that the perpendicular bisectors of the segments CD have a common point.

3. Prove that for every positive integer n there is a positive integer k such that the decimal representation of the number $2^n k$ consists of the digits 1 and 2 only.

Second Day

4. Seven students in a class compare their marks in the twelve subjects studied and observe that no two students have identical marks in all subjects. Show that one can choose six subjects such that the marks of any two students differ in at least one of these subjects.

5. Let ABC be an acute triangle. The circle with diameter AB intersects AC at E and BC at F. The tangents to the circle at E and F intersect at P. Prove that P lies on the altitude from C.

6. Let $a_1 \geq a_2 \geq \cdots \geq a_n \geq 0$ be real numbers with sum 1. Prove that
$$a_1^2 + 3a_2^2 + 5a_3^2 + \cdots + (2n-1)a_n^2 \leq 1.$$

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The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
Typed in LaTeX by Ercole Suppa
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