# Dutch Mathematical Olympiad 1995

## Second Round

September 15

1. A kangaroo jumps from lattice point to lattice point in the coordinate plane. It can make only two kinds of jumps: (A) 1 to the right and 3 up, and (B) 2 to the left and 4 down.

   (a) The start position of the kangaroo is $(0,0)$. Show that it can jump to the point $(19,95)$ and determine the number of jumps needed.

   (b) Show that if the start position is $(1,0)$, then it cannot reach $(19,95)$.

   (c) If the start position is $(0,0)$, find all points $(m,n)$ with $m,n \geq 0$ which the kangaroo can reach.

2. For any point $P$ on a segment $AB$, isosceles and right-angled triangles $AQP$ and $PRB$ are constructed on the same side of $AB$, with $AP$ and $PB$ as the bases. Determine the locus of the midpoint $M$ of $QR$ when $P$ describes the segment $AB$.

3. Let 101 marbles be numbered from 1 to 101. The marbles are divided over two baskets $A$ and $B$. The marble numbered 40 is in basket $A$. When this marble is removed from basket $A$ and put in $B$, the averages of the numbers in $A$ and $B$ both increase by $1/4$. How many marbles were there originally in basket $A$?

4. A number of spheres with radius 1 are being placed in the form of a square pyramid. First, there is a layer in the form of a square with $n^2$ spheres. On top of that layer comes the next layer with $(n-1)^2$ spheres, and so on. The top layer consists of only one sphere. Compute the height of the pyramid.

5. An array $(a_1, a_2, \ldots, a_{13})$ of 13 integers is called *tame* if for each $1 \leq i \leq 13$ the following condition holds: If $a_i$ is left out, the remaining twelve integers can be divided into two groups with the same sum of elements. A tame array is called *turbo tame* if the remaining twelve numbers can always be divided in two groups of six numbers having the same sum.

   (a) Give an example of a tame array of 13 integers (not all equal).

   (b) Prove that in a tame array all numbers are of the same parity.

   (c) Prove that in a turbo tame array all numbers are equal.




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