Dutch Mathematical Olympiad 1994

Second Round

September 16

1. A unit square is divided into two rectangles in such a way that the smaller rectangle can be put on the greater rectangle with every vertex of the smaller on exactly one of the edges of the greater. Calculate the dimensions of the smaller rectangle.

2. A sequence of integers $a_1, a_2, a_3, \dots$ is such that $a_1 = 2$, $a_2 = 3$, and

$$a_{n+1} = 2a_{n-1} \text{ or } 3a_n - 2a_{n-1} \quad \text{for all } n \geq 2.$$

   Prove that no number between 1600 and 2000 can be an element of the sequence.

3. (a) Prove that every multiple of 6 can be written as a sum of four cubes.

   (b) Prove that every integer can be written as a sum of five cubes.

4. Let P be a point on the diagonal BD of a rectangle $ABCD$, F be the projection of P on BC, and $H \neq B$ be the point on BC such that $BF = FH$. If lines PC and AH intersect at Q, prove that the areas of triangles APQ and CHQ are equal.

5. Three real numbers a, b, c satisfy the inequality $|ax^2 + bx + c| \leq 1$ for all $x \in [-1, 1]$. Prove that $|cx^2 + bx + a| \leq 2$ for all $x \in [-1, 1]$.

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