25-th Nordic Mathematical Contest

April 4, 2011

1. When $a_0, a_1, \dots, a_{1000}$ denote digits, can the sum of the 1001-digit numbers $a_0a_1\dots a_{1000}$ and $a_{1000}a_{999}\dots a_0$ have odd digits only?

2. In a triangle ABC assume $AB = AC$, and let D and E be points on the extension of segment BA beyond A and on the segment BC, respectively, such that the lines CD and AE are parallel. Prove $CD \geq \frac{4h}{BC} CE$, where h is the height from A in triangle ABC. When does equality hold?

3. Find all functions f such that
$$f(f(x)+y) = f(x^2-y) + 4yf(x)$$
for all real numbers x and y.

4. Show that for any integer $n \geq 2$ the sum of the fractions $\frac{1}{ab}$, where a and b are relatively prime positive integers such that $a < b \leq n$ and $a + b > n$, equals $\frac{1}{2}$.
(Integers a and b are called *relatively prime* if the greatest common divisor of a and b is 1.)

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