22-th Nordic Mathematical Contest

March 31, 2008

1. Determine all real numbers A, B, and C such that there exists a real function f that satisfies
$$f(x+f(y)) = Ax+By+C$$
for all real x and y.

2. Assume that $n \geq 3$ people with different names sit around a round table. We call any unordered pair of them, say M and N, dominating, if
   (i) M and N do not sit on adjacent seats, and
   (ii) on one (or both) of the arcs connecting M and N along the table edge, all people have names that come alphabetically after the names of M and N.

   Determine the minimal number of dominating pairs.

3. Let ABC be a triangle and let D and E be points on BC and CA, respectively, such that AD and GBE are angle bisectors of $\triangle ABC$. Let F and G be points on the circumcircle of $\triangle ABC$ such that $AF \| D$ and $FG \| BC$. Prove that
$$\frac{AG}{BG} = \frac{AB+AC}{AB+BC}.$$

4. The difference between the cubes of two consecutive positive integers is equal to n^2, where n is a positive integer. Show that n is the sum of two squares.

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