21-st Nordic Mathematical Contest

March 29, 2007

1. Find a solution to the equation $x^2 - 2x - 2007y^2 = 0$ in positive integers.

2. Three given rectangles cover the sides of a given triangle completely and each rectangle has a side parallel to a given line. Show that the rectangles also cover the interior of the triangle.

3. The number 10^{2007} is written on the blackboard. Anne and Berit play a two-player game in which the player in turn performs one of the following two operations:

   (i) replaces a number x on the blackboard by two integers a, b greater than 1 with $ab = x$;

   (ii) strikes off one or both of two equal numbers on the blackboard.

   The player who cannot perform any of the operations loses. Who wins if Anne starts and both players act in an optimal way?

4. A line through point A intersects a circle at points B and C with B between A and C. The two tangents from A touch the circle at points S and T. The lines ST and AC intersect at point P. Prove that

$$\frac{AP}{PC} = 2 \cdot \frac{AB}{BC}.$$