19-th Nordic Mathematical Contest

April 5, 2005

1. Find all positive integers k such that the product of the decimal digits of k equals $\frac{25}{8}k - 211$.

2. If a, b, c are positive numbers, prove the inequality

$$\frac{2a^2}{b+c} + \frac{2b^2}{c+a} + \frac{2c^2}{a+b} \geq a+b+c.$$

3. There are 2005 boys and girls sitting at a round table. No more than 668 of them are boys. A girl G is said to be in a *strong position* if, counting from G to either direction at any length (G herself included), the number of girls is always strictly larger than the number of boys. Prove that there always exists a girl in a strong position.

4. Circle $\mathscr{C}_1$ touches circle $\mathscr{C}_2$ internally at A. A line through A intersects $\mathscr{C}_1$ again at B and $\mathscr{C}_2$ again at C. The tangent to $\mathscr{C}_1$ at B intersects $\mathscr{C}_2$ at D and E. The tangents to $\mathscr{C}_1$ through C touch $\mathscr{C}_1$ at F and G. Prove that points D, E, F, G are concyclic.

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