18-th Nordic Mathematical Contest

April 1, 2004

1. Twenty-seven balls labelled from 1 to 27 are distributed in three bowls: red, blue, and yellow. What are the possible values of the number of balls in the red bowl if the average labels in the red, blue and yellow bowl are 15, 3, and 18, respectively?

2. The Fibonacci sequence is defined by $f_1 = 0$, $f_2 = 1$, and $f_{n+2} = f_{n+1} + f_n$ for $n \geq 1$. Prove that there is a strictly increasing arithmetic progression whose no term is in the Fibonacci sequence.

3. Given a finite sequence $x_{1,1}, x_{2,1}, \dots, x_{n,1}$ of integers ($n \geq 2$), not all equal, define the sequences $x_{1,k}, \dots, x_{n,k}$ by
$$x_{i,k+1} = \frac{1}{2}(x_{i,k} + x_{i+1,k}), \quad \text{where } x_{n+1,k} = x_{1,k}.$$
Show that if n is odd, then not all $x_{j,k}$ are integers. Is this also true for even n?

4. Let a, b, c be the sides and R be the circumradius of a triangle. Prove that
$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} \geq \frac{1}{R^2}.$$

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