17-th Nordic Mathematical Contest

April 3, 2003

1. The squares of a rectangular chessboard with 10 rows and 14 columns are colored alternatingly black and white in the usual manner. Some stones are placed the board (possibly more than one on the same square) so that there are an odd number of stones in each row and each column. Show that the total number of stones on black squares is even.

2. Find all triples (x,y,z) of integers satisfying the equation

$$x^3+y^3+z^3-3xyz=2003.$$

3. An interior point D of an equilateral triangle ABC is taken so that $\angle ADC=150^\circ$. Prove that the triangle whose sides are congruent to AD, BD and CD is right-angled.

4. Find all functions f from $\mathbb{R}\setminus\{0\}$ to itself satisfying

$$f(x)+f(y)=f(xyf(x+y))$$

for all $x,y\neq 0$ with $x+y\neq 0$.

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