14-th Nordic Mathematical Contest

March 30, 2000

1. In how many ways can the number 2000 be written as a sum of three positive, not necesarily different integers? (The order of summands is irrelevant.)

2. The persons $P_1, P_2, \ldots, P_n$ sit around a table in this order, and each one has a number of coins. Initially, P_1 has one coin more than P_2, P_2 has one coin more than P_3, etc. Now P_1 gives one coin to P_2, who in turn gives two coins to P_3, etc., up to P_n who gives n coins to P_1; then P_1 continues by giving $n+1$ coins to P_2, etc. The transactions go on until someone has not enough coins to give away one coin more than he just received. After this process ends, it turns out that there are two neighbors at the table one of whom has five times as many coins as the other. Find the number of persons and the number of coins circulating around the table.

3. In the triangle ABC, the bisectors of angles B and C meet the opposite sides at D and E, respectively. The bisectors intersect at point O such that $OD = OE$. Prove that either $\triangle ABC$ is isosceles or $\angle A = 60^\circ$.

4. A real function defined for $0 \le x \le 1$ satisfies $f(0) = 0$, $f(1) = 1$, and
$$\frac{1}{2} \le \frac{f(x) - f(y)}{f(y) - f(z)} \le 2$$
whenever $0 \le x < y < z \le 1$ and $z - y = y - x$. Show that $\frac{1}{7} \le f\left(\frac{1}{3}\right) \le \frac{4}{7}$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com