8-th Nordic Mathematical Contest

March 17, 1994

1. A point O is given in the interior of an equilateral triangle ABC with side length a. The lines AO, BO, CO meet the sides of the triangle at A_1, B_1, C_1. Prove that
$$OA_1 + OB_1 + OC_1 < a.$$

2. A finite set S of integer points in the coordinate plane is called a *two-neighbor set* if for each point $(p,q) \in S$, exactly two of the points $(p \pm 1, q), (p, q \pm 1)$ are in S. For which n does there exist a two-neighbor set consisting of n points?

3. A square sheet $ABCD$ is folded by placing corner D at a point D' on side BC. Let A' be the new position of A upon folding and let $A'D'$ intersect AB at E. Prove that the perimeter of triangle EBD' is half the perimeter of square $ABCD$.

4. Find all positive integers $n < 200$ for which $n^2 + (n+1)^2$ is a perfect square.

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