5-th Nordic Mathematical Contest

April 10, 1991

1. Define the last two digits of the number

$$2^5 + 2^{5^2} + 2^{5^3} + \cdots + 2^{5^{1991}},$$

when this number is written in a decimal system.

2. In a trapezoid $ABCD$ sides AB and CD are parallel and E is a fixed point on the side AB. Define a point F from a side CD so that the intersection of trianges ABF and CDE is as big as possible.

3. Prove that

$$\frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < \frac{2}{3}$$

for all $n \geq 2$.

4. Let $f(x)$ be a polynomial with integer coefficients. Let us assume that there exists a positive integer k and k consecutive integers $n, n+1, \ldots, n+k-1$ such that none of the numbers $f(n), f(n+1), \ldots, f(n+k-1)$ is divisible by k. Prove that the roots of $f(x)$ are not integers.

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