3-rd Nordic Mathematical Contest

April 10, 1989

1. Define the polynomial P with the least possible degree which satisfies the following conditions:

   (a) All coefficients of P are integers,

   (b) All roots of P are integers,

   (c) $P(0) = -1$,

   (d) $P(3) = 128$.

2. Three faces of a tetrahedron each have a right angle in the vertex where these faces meet. Areas of these three faces are A, B and C. Count the total area of the tetrahedron.

3. Let S be a group of all such values in the interval $[-1, 1]$, which have the property that for the series $x_0, x_1, x_2, \dots$, defined by equations $x_0 = t, x_{n+1} = 2x_n^2 - 1$, there exists a positive integer N such that $x_n = 1$ for each $n \geq N$. Prove that there are infinitely many values in the group S.

4. For which positive integers does the following hold: If $a_1, a_2, \dots, a_n$ are positive integers, $a_k \leq n$ for each k and $\sum_{k=1}^{n} = 2n$, then it is always possible to choose $a_{i_1}, a_{i_2}, \dots, a_{i_j}$ so that indexes $i_1, i_2, \dots, i_j$ are different numbers and $\sum_{k=1}^{j} a_{i_k} = n$?

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