1-st Nordic Mathematical Contest

March 30, 1987

1. Nine journalists, each from a different country, participates in a press conference. None of them can speak more than three languages, and each two journalists have at least one common language. Prove that at least five of the journalists can speak the same language.

2. Let $ABCD$ be a parallellogram in a plane. Let us draw two circles of radius R, one through the points A and B and another through the points B and C. Let E be another intersection point of these two circles. Let us assume that E is none of the vertices of the parallellogram. Prove that the radius of the circle passing through the points A, D and E is also R.

3. Let f be a function, defined for natural numbers, that is strictly increasing, such that values of the function are also natural numbers and which satisfies the conditions $f(2) = a > 2$ and $f(mn) = f(m)f(n)$ for all natural numbers m and n. Define the smallest possible value of a.

4. Let a, b and c be positive real numbers. Prove that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \leq \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2}.$$

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