11-th Mexican Mathematical Olympiad 1997.

Monterrey, Nuevo León

First Day

1. Determine all prime numbers p for which $8p^4 - 3003$ is a positive prime number.

2. In a triangle ABC, P and P' are points on side BC, Q on side CA, and R on side AB, such that $\frac{AR}{RB} = \frac{BP}{PC} = \frac{CQ}{QA} = \frac{CP'}{P'B}$. Let G be the centroid of triangle ABC and K be the intersection point of AP' and RQ. Prove that points P, G, K are collinear.

3. The numbers 1 through 16 are to be written in the cells of a 4×4 board.

   (a) Prove that this can be done in such a way that any two numbers in cells that share a side differ by at most 4.

   (b) Prove that this cannot be done in such a way that any two numbers in cells that share a side differ by at most 3.

Second Day

4. What is the minimum number of planes determined by 6 points in space which are not all coplanar, and among which no three are collinear?

5. Let P, Q, R be points on the sides Bc, CA, AB respectively of a triangle ABC. Suppose that BQ and CR meet at A', AP and CR meet at B', and AP and BQ meet at C', such that $AB' = B'C'$, $BC' = C'A'$, and $CA' = A'B'$. Compute the ratio of the area of $\triangle PQR$ to the area of $\triangle ABC$.

6. Prove that number 1 has infinitely many representations of the form
$$1 = \frac{1}{5} + \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n},$$
where n and a_i are positive integers with $5 < a_1 < a_2 < \cdots < a_n$.