2-nd Mediterranean Mathematical Competition 1999

1. Do there exist a circle and an infinite set of points on it such that the distance between any two of the points is rational?

2. A plane figure of area $A > n$ is given, where n is a positive integer. Prove that this figure can be placed onto a Cartesian plane so that it covers at least $n+1$ points with integer coordinates.

3. Let a, b, c be nonzero numbers and x, y, z be arbitrary positive numbers with $x+y+z=3$. Prove that inequality
$$\frac{3}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}} \geq \frac{x}{1+a^2}+\frac{y}{1+b^2}+\frac{z}{1+c^2}.$$

4. In a triangle ABC with $BC=a$, $CA=b$, $AB=c$ we have $\angle B = 4\angle A$. Show that
$$ab^2c^3 = (b^2-a^2-ac)\left((a^2-b^2)^2-a^2c^2\right).$$

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