## 10-th Mediterranean Mathematical Competition 2007

- 1. Let  $x \le y \le z$  be real numbers such that xy + yz + zx = 1. Prove that  $xz < \frac{1}{2}$ . Is it possible to improve the value of constant  $\frac{1}{2}$ ?
- 2. The diagonals *AC* and *BD* of a convex cyclic quadrilateral *ABCD* intersect at point *E*. Given that AB = 39, AE = 45, AD = 60 and BC = 56, determine the length of *CD*.
- 3. In the triangle *ABC*, the angle  $\alpha = \angle BAC$  and the side a = BC are given. Assume that  $a = \sqrt{rR}$ , where *r* is the inradius and *R* the circumradius. Compute all possible lengths of sides *AB* and *AC*.
- 4. Let x > 1 be a non-integer number. Prove that

$$\left(\frac{x+\{x\}}{[x]} - \frac{[x]}{x+\{x\}}\right) + \left(\frac{x+[x]}{\{x\}} - \frac{\{x\}}{x+[x]}\right) > \frac{9}{2}$$



The IMO Compendium Group, D. Djukić, V. Janković, I. Matić, N. Petrović Provided by Salem Malikić www.imomath.com