8-th Mediterranean Mathematical Competition 2005

1. The professor told Petar the product of two natural numbers, and told Marko their sum. Then one of the boys told the other: ”There is no way for you to determine my number.” The other boy responded: ”You are wrong, your number is 136.” Find the numbers the professor told each boy.

2. Two circles k and k' have the common center O and radii r and r' respectively. A ray Ox meets k at A, while its complementary ray Ox' meets k' at B. Another ray Ot meets k at E and k' at F. Prove that the circles OAE, OBF and the circles with diameters EF and AB all pass through a single point.

3. Let $A_1,\dots,A_n$ ($n \geq 3$) be finite sets of natural numbers. Prove that
$$\frac{1}{n}\sum_{i=1}^{n}|A_i| + \frac{1}{\binom{n}{3}}\sum_{i<j<k}|A_i \cap A_j \cap A_k| \geq \frac{2}{\binom{n}{2}}\sum_{i<j}|A_i \cap A_j|.$$

4. Let A be the set of cubic polynomials $f(x)$ with the leading coefficient 1 having the following property: There exist a prime number p not dividing 2004 and a positive integer q coprime to p and 2004 such that $f(p) = 2004$ and $f(q) = 0$. Show that there is an infinite subset $B \subset A$ such that the graphs of all polynomials from B are identic up to a translation.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com