4-th Mediterranean Mathematical Competition 2001

1. Let P and Q be points on a circle k. A chord AC of k passes through the midpoint M of PQ. Consider a trapezoid $ABCD$ inscribed in k with $AB \parallel CD$. Prove that the intersection point X of AD and BC depends only on k and P,Q.

2. Find al integers n for which the polynomial $p(x) = x^5 - nx - n - 2$ can be represented as a product of two non-constant polynomials with integer coefficients.

3. Show that there exists a positive integer N such that the decimal representation of 2000^N starts with the digits 200120012001.

4. Let $\mathscr{S}$ be the set of points inside a given equilateral triangle ABC with side 1 or on its boundary. For any $M \in \mathscr{S}$, a_M, b_M, c_M denote the distances from M to BC, CA, AB, respectively. Define
$$f(M) = a_M^3(b_M - c_M) + b_M^3(c_M - a_M) + c_M^3(a_M - b_M).$$
   (a) Describe the set $\{M \in \mathscr{S} \mid f(M) \geq 0\}$ geometrically.
   (b) Find the minimum and maximum values of $f(M)$ as well as the points in which these are attained.

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