6-th Macedonian Mathematical Olympiad 1999

1. In a set of 21 real numbers, the sum of any 10 numbers is less than the sum of the remainiong 11 numbers. Prove that all the numbers are positive.

2. We are given 13 apparently equal balls, all but one having the same weight (the remaining one has a different weight). Is it posible to determine the ball with the different weight in 3 weighings?

3. Let the two tangents from a point A outside a circle k touch k at M and N. A line p through A intersects k at B and C, and D is the midpoint of MN. Prove that MN bisects the angle BDC.

4. Do there exist 100 straight lines on a plane such that they intersect each other in exactly 1999 points?

5. If a,b,c are positive numbers with $a^2+b^2+c^2=1$, prove that

$$a+b+c+\frac{1}{abc}\geq 4\sqrt{3}.$$

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