3-rd Macedonian Mathematical Olympiad 1996

1. Let $ABCD$ be a parallelogram which is not a rectangle and E be the point in its plane such that $AE \perp AB$ and $CE \perp CB$. Prove that $\angle DEA = \angle CEB$.

2. Let $\mathscr{P}$ be the set of all polygons in the plane and let $M : \mathscr{P} \to \mathbb{R}$ be a mapping that satisfies:

   (i) $M(P) \geq 0$ for each polygon P;

   (ii) $M(P) = x^2$ if P is an equilateral triangle of side x;

   (iii) If a polygon P is partitioned into polygons S and T, then $M(P) = M(S) + M(T)$;

   (iv) If polygons P and T are congruent, then $M(P) = M(T)$.

   Determine $M(P)$ if P is a rectangle with edges x and y.

3. Prove that if α, β, γ are angles of a triangle, then

$$\frac{1}{\sin\alpha} + \frac{1}{\sin\beta} \geq \frac{8}{3 + 2\cos\gamma}.$$

4. A polygon is called *good* if it satisfies the following conditions:

   (i) All its angles are in $(0, \pi)$ or in $(\pi, 2\pi)$;

   (ii) It is not self-intersecing;

   (iii) For any three sides, two are parallel and equal.

   Find all n for which there exists a good n-gon.

5. Find the greatest n for which there exist n lines in space, passing through a single point, such that any two of them form the same angle.

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