7-th Macedonian Mathematical Olympiad 2000

1. Let AB be a diameter of a circle with center O, and CD be a chord perpendicular to AB. A chord AE intersect CO at M, while DE and BC intersect at N. Prove that $CM : CO = CN : CB$.

2. If $a_1, a_2, \dots, a_n$ are positive numbers, find the maximum value of
$$\frac{a_1 a_2 \cdots a_{n-1} a_n}{(1+a_1)(a_1+a_2)\cdots(a_{n-1}+a_n)(a_n+2^{n+1})}.$$

3. In a triangle with sides a, b, c, t_a, t_b, t_c are the corresponding medians and D the diameter of the circumcircle. Prove that
$$\frac{a^2+b^2}{t_c}+\frac{b^2+c^2}{t_a}+\frac{c^2+a^2}{t_b} \le 6D.$$

4. Let a, b be coprime positive integers. Show that the number of positive integers n for which the equation $ax+by=n$ has no positive integer solutions is equal to $\frac{(a-1)(b-1)}{2}-1$.

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