7-th Japanese Mathematical Olympiad 1997

Final Round – February

1. Prove that among any ten points inside a circle of diameter 5 there exist two whose distance is less than 2.

2. Let a,b,c be positive integers. Prove the inequality
$$\frac{(b+c-a)^2}{(b+c)^2+a^2}+\frac{(c+a-b)^2}{(c+a)^2+b^2}+\frac{(a+b-c)^2}{(a+b)^2+c^2}\geq\frac{3}{5}$$
and determine when the equality holds.

3. Let $\mathscr{G}$ be a simple graph with 9 vertices. Assume that among any five vertices there are two vertices and a sequence of edges $e_1,\dots,e_n$ forming a path between the two points. What is the minimum possible number of edges in the graph $\mathscr{G}$?

4. Let A,B,C,D be points in space which are not on a plane. Suppose that $f(X)=AX+BX+CX+DX$ attains its minimum at a point $X=X_0$ distinct from A,B,C,D. Prove that $\angle AX_0B=\angle CX_0D$.

5. Let n be a positive integer. Prove that to each vertex of a regular 2^n-gon one can assign a letter A or B in such a way that all the sequences of n letters which appear on this 2^n-gon as an arc directed clockwise from some vertex are mutually distinct.

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