# Japanese Mathematical Olympiad 1992

## Final Round

1. Let $x, y$ be coprime positive integers with $xy > 1$, and let $n$ be an even positive integer. Prove that $x^n + y^n$ is not divisible by $x + y$.

2. Let $ABC$ be a given triangle with the area 1. Let $D$ and $E$ be points on sides $AB$ and $AC$ respectively. Lines $BE$ and $CD$ intersect at $P$. Find the maximum possible value of $S_{PDE}$ under the condition $S_{BCED} = 2S_{PBC}$.

3. For every positive integer $n \geq 2$ prove that $\sum_{k=1}^{n-1} \frac{n}{n-k} \cdot \frac{1}{2k-1} < 4$.

4. Comsider an $m \times n$ matrix $A$ satisfying the following conditions:

   (1) $m \leq n$;

   (2) the entries of $A$ are zeros and ones;

   (3) whenever $f : \{1, \dots, m\} \to \{1, \dots, n\}$ is an injection, it holds that $A_{i,f(i)} = 0$ for some $i$.

   Prove that there exist $S \subset \{1, \dots, m\}$ and $T \subset \{1, \dots, n\}$ such that

   (i) $A_{ij} = 0$ for every $i \in S$, $j \in T$;

   (ii) $|S| + |T| > n$.

5. Let $a_1, a_2, a_3, a_4$ be positive integers coprime to $n \in \mathbb{N}$. Suppose that

   $$(ka_1)_n + (ka_2)_n + (ka_3)_n + (ka_4)_n = 2n \qquad \text{for all } k = 1, 2, \dots, n-1,$$

   where $(a)_n = a - n[\frac{a}{n}]$. Prove that $(a_1)_n + (a_j)_n = n$ for some $j \in \{2, 3, 4\}$.



The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com