# Italian IMO Team Selection Test 1993

*Cortona, May 22, 1993*

Time allowed: 4 hours

1. Let $x_1, x_2, \ldots, x_n$ ($n \geq 2$) be positive numbers with the sum 1. Prove that
$$\sum_{i=1}^{n} \frac{1}{\sqrt{1-x_i}} \geq n\sqrt{\frac{n}{n-1}}.$$

2. Suppose that $p, q$ are prime numbers such that
$$\sqrt{p^2+7pq+q^2}+\sqrt{p^2+14pq+q^2}$$
is an intger. Show that $p = q$.

3. Let $ABC$ be an isosceles triangle with base $AB$ and $D$ be a point on side $AB$ such that the incircle of triangle $ACD$ is congruent to the excircle of triangle $DCB$ across $C$. Prove that the diameter of each of these circles equals half the altitude of $\triangle ABC$ from $A$.

4. An $m \times n$ chessboard with $m, n \geq 2$ is given. Some dominoes are placed on the chessboard so that the following conditions are satisfied:

   (i) Each domino occupies two adjacent squares of the chessboard;

   (ii) It is not possible to put another domino onto the chessboard without overlapping;

   (iii) It is not possible to slide a domino horizontally or vertically without overlapping.

   Prove that the number of squares that are not covered by a domino is less than $\frac{1}{5}mn$.



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