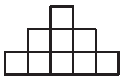
11-th Italian Mathematical Olympiad 1995

Cesenatico, May 5, 1995

1. Determine for which values of n it is possible to tile a square of side n with figures of the type shown in the picture.



2. No two of 20 students in a class have the same scores on both written and oral examinations in mathematics. We say that student A is better than B if his two scores are greater than or equal to the corresponding scores of B. The scores are integers between 1 and 10.

   (a) Show that there exist three students A, B, C such that A is better than B and B is better than C.

   (b) Would the same be true for a class of 19 students?

3. In a town there are four pubs, A, B, C, D, and any two of them are connected to each other except A and D. A drunkard wanders about the pubs starting with A and, after having a drink, goes to any of the pubs directly connected, with equal probability.

   (a) What is the probability that the drunkard is at C at its fifth drink?

   (b) Where is the drunkard most likely to be after n drinks ($n > 5$)?

4. An acute-angled triangle ABC is inscribed in a circle with center O. The bisector of $\angle A$ meets BC at D, and the perpendicular to AO through D meets the segment AC in a point P. Show that $AB = AP$.

5. Two non-coplanar circles in space are tangent at a point and have the same tangents at this point. Show that both circles lie on some sphere.

6. Find all pairs of positive integers x, y such that $x^2 + 615 = 2^y$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com