14-th Iranian Mathematical Olympiad 1996/1997

First Round

1. Let a, b, c be real numbers. Prove that there exists a triangle with side lengths a, b, c if and only if

$$2(a^4 + b^4 + c^4) < (a^2 + b^2 + c^2)^2.$$

2. Prove that if a, b, c, d are positive integers such that $ad = bc$, then $a + b + c + d$ cannot be a prime number.

3. Let N be the midpoint of side BC of triangle ABC. Right isosceles triangles ABM and ACP are constructed outside the triangle, with bases AB and AC. Prove that $\triangle MNP$ is also a right isosceles triangle.

4. Let n blue points A_i and n red points B_i ($i = 1, 2, \ldots, n$) be situated on a line. Prove that

$$\sum_{i,j} A_i B_j \geq \sum_{i<j} A_i A_j + \sum_{i<j} B_i B_j.$$