12-th Indian Mathematical Olympiad 1997

1. A line through the vertex C of a parallelogram $ABCD$ meets the extensions of sides AB and AD at E and F respectively. Prove that

$$AC^2 + CE \cdot CF = AB \cdot AE + AD \cdot AF.$$

2. Show that there do not exist positive integers m and n such that

$$\frac{m}{n} + \frac{n+1}{m} = 4.$$

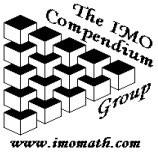
3. Suppose that a, b, c are distinct real numbers and t a real number such that $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a} = t$. Show that $abc + t = 0$.

4. One hundred rays emanating from the center of a square divide the square into 100 parts, all having equal parameter p. Show that $1.4 < p < 1.5$.

5. Find the number of 4×4 arrays tables whose entries are from the set $\{0, 1, 2, 3\}$ such that the sum of the numbers in each of the four rows and in each of the four columns is divisible by 4.

6. Let a and b be positive numbers for which the cubic equation $x^3 - ax + b = 0$ has three (not necessarily distinct) real roots. If α is the one with minimal absolute value, prove that

$$\frac{b}{a} < \alpha < \frac{3b}{2a}.$$



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