# 11-th Iberoamerican Mathematical Olympiad

## San Hose, Costa Rica, September 22–30, 1996

*First Day – September 24*

1. Let $n$ be a natural number. A cube of side $n$ can be split into 1996 cubes of natural side length. Find the minimum possible value of $n$. *(Brazil)*

2. Let $M$ be the midpoint of the median $AD$ of a triangle $ABC$. Line $BM$ meets side $AC$ at $N$. Prove that $AB$ is tangent to the circumcircle of triangle $NBC$ if and only if the equality $\frac{BM}{MN} = \frac{BC^2}{BN^2}$ holds. *(Spain)*

3. We have a chessboard of size $(k^2-k+1)\times(k^2-k+1)$, where $k-1=p$ is a prime number. For each prime $p$, give a method of distribution of the numbers 0 and 1, one number in each square of the chessboard, in such a manner that in each row or column there are exactly $k$ zeros, and no rectangle with sides parallel to the sides of the chessboard has zeros on the vertices. *(Spain)*

*Second Day – September 25*

4. Given a natural number $n \geq 2$, all the fractions of the form $\frac{1}{ab}$, with $a$ and $b$ coprime positive integers with $a < b \leq n$ and $a+b > n$ are considered. Prove that the sum of all these fractions equals $\frac{1}{2}$. *(Brazil)*

5. Three coins, $A, B, C$ are situated one at each vertex of an equilateral triangle of side $n$. The triangle is divided into small equilateral triangles of side 1 by lines parallel to the sides. Initially, all the lines of the figure are blue. The coins move along the lines, painting in red their trajectory, following the two rules:

   (i) First coin to move is $A$, then $B$, then $C$, then again $A$, and so on. At each turn, a coin paints exactly one side of one of the small triangles.

   (ii) A coin can not move along a segment that is already painted red, but it can stay at an endpoint of a red segment, not necessarily alone.

   Show that for all integers $n > 0$ it is possible to paint all the sides of all the small triangles red. *(Peru)*

6. Let $A_1, A_2, \dots, A_n$ be distinct points in the plane. Suppose that each point $A_i$ can be assigned a real number $\lambda_i \neq 0$ in such a way that

$$A_iA_j^2 = \lambda_i + \lambda_j, \quad \text{for all } i, j \text{ with } i \neq j.$$

   (a) Show that $n \leq 4$.

   (b) Prove that if $n = 4$, then $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \frac{1}{\lambda_4} = 0$. *(Spain)*