1-st Iberoamerican Mathematical Olympiad

Paipa – Villa de Leyva, Colombia, December 9–15, 1985

First Day

1. Find all triples of integers (a,b,c) such that
$$\begin{aligned} a+b+c &= 24, \\ a^2+b^2+c^2 &= 210, \\ abc &= 440. \end{aligned}$$

2. A point P inside an equilateral triangle ABC satisfies $PA=5$, $PB=7$, and $PC=8$. Find the side length of the triangle ABC.

3. Find the roots r_1,r_2,r_3,r_4 of the equation $4x^4-ax^3+bx^2-cx+5=0$, assuming that they are positive real numbers and that
$$\frac{r_1}{2}+\frac{r_2}{4}+\frac{r_3}{5}+\frac{r_4}{8}=1.$$

Second Day

4. Suppose that the real numbers x,y,z are different from 1 and satisfy
$$\frac{yz-x^2}{1-x}=\frac{zx-y^2}{1-y}=\frac{xy-z^2}{1-z}\,.$$
Show that each of these fractions equals $x+y+z$.

5. A function f maps the positive integers to the nonnegative integers such that
   (i) $f(rs)=f(r)+f(s)$ for all r,s,
   (ii) $f(n)=0$ if the last digit of n is 3,
   (iii) $f(10)=0$.

   Find $f(1985)$ and justify your answer.

6. Let O be the center and r the radius of the circumcircle of a triangle ABC. The lines AO,BO,CO meet the opposite sides of the triangle at D,E,F, respectively. Prove that
$$\frac{1}{AD}+\frac{1}{BE}+\frac{1}{CF}=2r.$$

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