32-nd International Mathematical Olympiad

Sigtuna, Sweden, July 12–23, 1991

First Day – July 17

1. Prove for each triangle ABC the inequality
$$\frac{1}{4} < \frac{IA \cdot IB \cdot IC}{l_A l_B l_C} \le \frac{8}{27},$$
where I is the incenter and l_A, l_B, l_C are the lengths of the angle bisectors of ABC.
(Soviet Union)

2. Let $n > 6$ and let $a_1 < a_2 < \dots < a_k$ be all natural numbers that are less than n and relatively prime to n. Show that if $a_1, a_2, \dots, a_k$ is an arithmetic progression, then n is a prime number or a natural power of two. *(Romania)*

3. Let $S = \{1, 2, 3, \dots, 280\}$. Find the minimal natural number n such that in any n-element subset of S there are five numbers that are pairwise relatively prime. *(China)*

Second Day – July 18

4. Suppose G is a connected graph with n edges. Prove that it is possible to label the edges of G from 1 to n in such a way that in every vertex v of G with two or more incident edges, the set of numbers labeling those edges has no common divisor greater than 1. *(United States of America)*

5. Let ABC be a triangle and M an interior point in ABC. Show that at least one of the angles $\measuredangle MAB, \measuredangle MBC$, and $\measuredangle MCA$ is less than or equal to 30°. *(France)*

6. Given a real number $a > 1$, construct an infinite and bounded sequence $x_0, x_1, x_2, \dots$ such that for all natural numbers i and j, $i \neq j$, the following inequality holds:
$$|x_i - x_j||i - j|^a \ge 1.$$
(Netherlands)

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