30-th International Mathematical Olympiad

Braunschweig – Niedersachen, FR Germany, July 13–24, 1989

First Day – July 18

1. Prove that the set $\{1,2,\dots,1989\}$ can be expressed as the disjoint union of 17 subsets $A_1,A_2,\dots,A_{17}$ such that:

   (i) each A_i contains the same number of elements;

   (ii) the sum of all elements of each A_i is the same for $i=1,2,\dots,17$.

   (Philippines)

2. Let ABC be a triangle. The bisector of angle A meets the circumcircle of triangle ABC in A_1. Points B_1 and C_1 are defined similarly. Let AA_1 meet the lines that bisect the two external angles at B and C in point A^0. Define B^0 and C^0 similarly. If $S_{X_1X_2\dots X_n}$ denotes the area of the polygon $X_1X_2\dots X_n$, prove that

   $$S_{A^0B^0C^0}=2S_{AC_1BA_1CB_1}\geq 4S_{ABC}.$$

   (Australia)

3. Given a set S in the plane containing n points and satisfying the conditions

   (i) no three points of S are collinear,

   (ii) for every point P of S there exist at least k points in S that have the same distance to P,

   prove that the following inequality holds:

   $$k<\frac{1}{2}+\sqrt{2n}.$$

   (Netherlands)

Second Day – July 19

4. The quadrilateral $ABCD$ has the following properties:

   (i) $AB=AD+BC$;

   (ii) there is a point P inside it at a distance x from the side CD such that $AP=x+AD$ and $BP=x+BC$.

   Show that

   $$\frac{1}{\sqrt{x}}\geq\frac{1}{\sqrt{AD}}+\frac{1}{\sqrt{BC}}.$$

   (Iceland)

5. For which positive integers n does there exist a positive integer N such that none of the integers $1+N,2+N,\dots,n+N$ is the power of a prime number? *(Sweden)*

6. We consider permutations $(x_1,\dots,x_{2n})$ of the set $\{1,\dots,2n\}$ such that $|x_i-x_{i+1}|=n$ for at least one $i\in\{1,\dots,2n-1\}$. For every natural number n, find out whether permutations with this property are more or less numerous than the remaining permutations of $\{1,\dots,2n\}$. *(Poland)*

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