# 26-th International Mathematical Olympiad

## Joutsa, Finland, June 29–July 11, 1985

*First Day – July 4*

1. A circle whose center is on the side $ED$ of the cyclic quadrilateral $BCDE$ touches the other three sides. Prove that $EB+CD=ED$.

   *(Great Britain)*

2. Each of the numbers in the set $N=\{1,2,3,\dots,n-1\}$, where $n\geq 3$, is colored with one of two colors, say red or black, so that:

   (i) $i$ and $n-i$ always receive the same color, and

   (ii) for some $j\in N$ relatively prime to $n$, $i$ and $|j-i|$ receive the same color for all $i\in N$, $i\neq j$.

   Prove that all numbers in $N$ must receive the same color. *(Australia)*

3. The weight $w(p)$ of a polynomial $p$, $p(x)=\sum_{i=0}^{n}a_ix^i$, with integer coefficients $a_i$ is defined as the number of its odd coefficients. For $i=0,1,2,\dots$, let $q_i(x)=(1+x)^i$. Prove that for any finite sequence $0\leq i_1<i_2<\cdots<i_n$ the inequality

   $$w(q_{i_1}+\cdots+q_{i_n})\geq w(q_{i_1})$$

   holds. *(Netherlands)*

*Second Day – July 5*

4. Given a set $M$ of 1985 positive integers, none of which has a prime divisor larger than 26, prove that $M$ has four distinct elements whose geometric mean is an integer. *(Mongolia)*

5. A circle with center $O$ passes through points $A$ and $C$ and intersects the sides $AB$ and $BC$ of the triangle $ABC$ at points $K$ and $N$, respectively. The circumscribed circles of the triangles $ABC$ and $KBN$ intersect at two distinct points $B$ and $M$. Prove that $\measuredangle OMB=90^\circ$. *(Soviet Union)*

6. The sequence $f_1,f_2,\dots,f_n,\dots$ of functions is defined for $x>0$ recursively by

   $$f_1(x)=x,\qquad f_{n+1}(x)=f_n(x)\left(f_n(x)+\frac{1}{n}\right).$$

   Prove that there exists one and only one positive number $a$ such that $0<f_n(a)<f_{n+1}(a)<1$ for all integers $n\geq 1$. *(Sweden)*



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