22-nd International Mathematical Olympiad

Washington DC, United States of America, July 8–20, 1981

First Day – July 13

1. Find the point P inside the triangle ABC for which

$$\frac{BC}{PD}+\frac{CA}{PE}+\frac{AB}{PF}$$

is minimal, where PD, PE, PF are the perpendiculars from P to BC, CA, AB respectively. *(Great Britain)*

2. Let $f(n,r)$ be the arithmetic mean of the minima of all r-subsets of the set $\{1,2,\ldots,n\}$. Prove that $f(n,r)=\dfrac{n+1}{r+1}$. *(FR Germany)*

3. Determine the maximum value of m^2+n^2 where m and n are integers satisfying

$$m,n\in\{1,2,\ldots,1981\} \quad \text{and} \quad (n^2-mn-m^2)^2=1.$$

(Netherlands)

Second Day – July 14

4. (a) For which values of $n>2$ is there a set of n consecutive positive integers such that the largest number in the set in the set is a divisor of the least common multiple of the remaining $n-1$ numbers?

   (b) For which values of $n>2$ is there a unique set having the stated property? *(Belgium)*

5. Three equal circles touch the sides of a triangle and have one common point O. Show that the center of the circle inscribed in and of the circle circumscribed about the triangle ABC and the point O are collinear.

(Soviet Union)

6. Assume that $f(x,y)$ is defined for all positive integers x and y, and that the following equations are satisfied:

$$\begin{aligned} f(0,y) &= y+1, \\ f(x+1,0) &= f(x,1), \\ f(x+1,y+1) &= f(x,f(x+1,y)). \end{aligned}$$

Determine $f(4,1981)$. *(Finland)*

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