21-st International Mathematical Olympiad

London, United Kingdom, 1979

First Day – July 2

1. Given that $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots-\frac{1}{1318}+\frac{1}{1319}=\frac{p}{q}$, where p and q are natural numbers having no common factor, prove that p is divisible by 1979. *(FR Germany)*

2. A pentagonal prism $A_1A_2\dots A_5B_1B_2\dots B_5$ is given. The edges, the diagonals of the lateral walls, and the internal diagonals of the prism are each colored either red or green in such a way that no triangle whose vertices are vertices of the prism has its three edges of the same color. Prove that all edges of the bases are of the same color. *(Bulgaria)*

3. There are two circles in the plane. Let a point A be one of the points of intersection of these circles. Two points begin moving simultaneously with constant speeds from the point A, each point along its own circle. The two points return to the point A at the same time.
   Prove that there is a point P in the plane such that at every moment of time the distances from the point P to the moving points are equal.
   (Soviet Union)

Second Day – July 3

4. Given a point P in a given plane π and also a given point Q not in π, determine all points R in π such that $\frac{QP+PR}{QR}$ is a maximum.
   (United States of America)

5. The nonnegative real numbers x_1,x_2,x_3,x_4,x_5,a satisfy the following relations:
$$\sum_{i=1}^{5} ix_i=a,\qquad \sum_{i=1}^{5} i^3x_i=a^2,\qquad \sum_{i=1}^{5} i^5x_i=a^3.$$
   What are the possible values of a? *(Israel)*

6. Let S and F be two opposite vertices of a regular octagon. A counter starts at S and each second is moved to one of the two neighboring vertices of the octagon. The direction is determined by the toss of a coin. The process ends when the counter reaches F. We define a_n to be the number of distinct paths of duration n seconds that the counter may take to reach F from S. Prove that for $n=1,2,3,\dots$,
$$a_{2n-1}=0,\qquad a_{2n}=\frac{1}{\sqrt{2}}(x^{n-1}-y^{n-1}),\qquad \text{where } x=2+\sqrt{2},\ y=2-\sqrt{2}.$$
   (FR Germany)

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