20-th International Mathematical Olympiad

Bucharest, Romania, 1978

First Day – July 6

1. Let $n > m \geq 1$ be natural numbers such that the groups of the last three digits in the decimal representation of $1978^m, 1978^n$ coincide. Find the ordered pair (m,n) of such m,n for which $m+n$ is minimal. *(Cuba)*

2. Given any point P in the interior of a sphere with radius R, three mutually perpendicular segments PA, PB, PC are drawn terminating on the sphere and having one common vertex in P. Consider the rectangular parallelepiped of which PA, PB, PC are coterminal edges. Find the locus of the point Q that is diagonally opposite P in the parallelepiped when P and the sphere are fixed. *(United States of America)*

3. Let $\{f(n)\}$ be a strictly increasing sequence of positive integers: $0 < f(1) < f(2) < f(3) < \dots$. Of the positive integers not belonging to the sequence, the nth in order of magnitude is $f(f(n))+1$. Determine $f(240)$. *(Great Britain)*

Second Day – July 7

4. In a triangle ABC we have $AB = AC$. A circle is tangent internally to the circumcircle of ABC and also to the sides AB, AC, at P, Q respectively. Prove that the midpoint of PQ is the center of the incircle of ABC. *(United States of America)*

5. Let $\varphi : \{1,2,3,\dots\} \to \{1,2,3,\dots\}$ be injective. Prove that for all n,
$$\sum_{k=1}^{n} \frac{\varphi(k)}{k^2} \geq \sum_{k=1}^{n} \frac{1}{k}.$$
(France)

6. An international society has its members in 6 different countries. The list of members contains 1978 names, numbered $1, 2, \dots, 1978$. Prove that there is at least one member whose number is the sum of the numbers of two, not necessarily distinct, of his compatriots. *(Netherlands)*

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