16-th International Mathematical Olympiad

Erfurt – Berlin, DR Germany, July 4–17, 1974

First Day – July 8

1. Alice, Betty, and Carol took the same series of examinations. There was one grade of A, one grade of B, and one grade of C for each examination, where A, B, C are different positive integers. The final test scores were

Alice	Betty	Carol
20	10	9

If Betty placed first in the arithmetic examination, who placed second in the spelling examination? *(United States of America)*

2. Let $\triangle ABC$ be a triangle. Prove that there exists a point D on the side AB such that CD is the geometric mean of AD and BD if and only if

$$\sqrt{\sin A \sin B} \le \sin \frac{C}{2}.$$

(Finland)

3. Prove that there does not exist a natural number n for which the number

$$\sum_{k=0}^{n} \binom{2n+1}{2k+1} 2^{3k}$$

is divisible by 5. *(Romania)*

Second Day – July 9

4. Consider a partition of an 8×8 chessboard into p rectangles whose interiors are disjoint such that each rectangle contains an equal number of white and black cells. Assume that $a_1 < a_2 < \cdots < a_p$, where a_i denotes the number of white cells in the ith rectangle. Find the maximal p for which such a partition is possible and for that p determine all possible corresponding sequences $a_1, a_2, \ldots, a_p$. *(Bulgaria)*

5. If a, b, c, d are arbitrary positive real numbers, find all possible values of

$$S = \frac{a}{a+b+d} + \frac{b}{a+b+c} + \frac{c}{b+c+d} + \frac{d}{a+c+d}.$$

(Netherlands)

6. Let $P(x)$ be a polynomial with integer coefficients. If $n(P)$ is the number of (distinct) integers k such that $P^2(k) = 1$, prove that $n(P) - \deg(P) \le 2$, where $\deg(P)$ denotes the degree of the polynomial P. *(Sweden)*

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