14-th International Mathematical Olympiad

Warsaw – Toruna, Poland, July 5–17, 1972

First Day – July 10

1. A set of 10 positive integers is given such that the decimal expansion of each of them has two digits. Prove that there are two disjoint subsets of the set with equal sums of their elements. *(Soviet Union)*

2. Prove that for each $n \geq 4$ every cyclic quadrilateral can be decomposed into n cyclic quadrilaterals. *(Netherlands)*

3. Let m and n be nonnegative integers. Prove that $\dfrac{(2m)!(2n)!}{m!n!(m+n)!}$ is an integer $(0! = 1)$. *(Great Britain)*

Second Day – July 11

4. Find all solutions in positive real numbers x_i $(i = 1,2,3,4,5)$ of the following system of inequalities:
$$\begin{aligned}(x_1^2 - x_3x_5)(x_2^2 - x_3x_5) &\leq 0\\ (x_2^2 - x_4x_1)(x_3^2 - x_4x_1) &\leq 0\\ (x_3^2 - x_5x_2)(x_4^2 - x_5x_2) &\leq 0\\ (x_4^2 - x_1x_3)(x_5^2 - x_1x_3) &\leq 0\\ (x_5^2 - x_2x_4)(x_1^2 - x_2x_4) &\leq 0.\end{aligned}$$
(Netherlands)

5. Let f and φ be real functions defined in the interval $(-\infty,\infty)$ satisfying the functional equation
$$f(x+y) + f(x-y) = 2\varphi(y)f(x),$$
for arbitrary real x,y (give examples of such functions). Prove that if $f(x)$ is not identically 0 and $|f(x)| \leq 1$ for all x, then $|\varphi(x)| \leq 1$ for all x.
(Bulgaria)

6. Given four distinct parallel planes, show that a regular tetrahedron exists with a vertex on each plane. *(Great Britain)*

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