11-th International Mathematical Olympiad

Bucharest, Romania, July 5–20, 1969

First Day – July 10

1. Prove that there exist infinitely many natural numbers a with the following property: the number $z = n^4 + a$ is not prime for any natural number n.
(DR Germany)

2. Let $a_1, a_2, \dots, a_n$ be real constants and
$$y(x) = \cos(a_1 + x) + \frac{\cos(a_2 + x)}{2} + \frac{\cos(a_3 + x)}{2^2} + \cdots + \frac{\cos(a_n + x)}{2^{n-1}}.$$
If x_1, x_2 are real and $y(x_1) = y(x_2) = 0$, prove that $x_1 - x_2 = m\pi$ for some integer m.
(Hungary)

3. Find conditions on the positive real number a such that there exists a tetrahedron k of whose edges ($k = 1,2,3,4,5$) have length a, and the other $6 - k$ edges have length 1.
(Poland)

Second Day – July 11

4. Let AB be a diameter of a circle γ. A point C different from A and B is on the circle γ. Let D be the projection of the point C onto the line AB. Consider three other circles γ_1, γ_2, and γ_3 with the common tangent AB: γ_1 inscribed in the triangle ABC, and γ_2 and γ_3 tangent to both (the segment) CD and γ. Prove that γ_1, γ_2, and γ_3 have two common tangents.
(Netherlands)

5. Given n points in the plane such that no three of them are collinear, prove that one can find at least $\binom{n-3}{2}$ convex quadrilaterals with their vertices at these points.
(Mongolia)

6. Under the conditions $x_1, x_2 > 0$, $x_1 y_1 > z_1^2$, and $x_2 y_2 > z_2^2$, prove the inequality
$$\frac{8}{(x_1 + x_2)(y_1 + y_2) - (z_1 + z_2)^2} \le \frac{1}{x_1 y_1 - z_1^2} + \frac{1}{x_2 y_2 - z_2^2}.$$
(Soviet Union)

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