4-th International Mathematical Olympiad

Prague – Hluboka, Czechoslovakia, July 7–15, 1962

First Day

1. Find the smallest natural number n with the following properties:

   (a) In decimal representation it ends with 6.

   (b) If we move this digit to the front of the number, we get a number 4 times larger. *(Poland)*

2. Find all real numbers x for which
$$\sqrt{3-x}-\sqrt{x+1}>\frac{1}{2}\,.$$ *(Hungary)*

3. A cube $ABCDA'B'C'D'$ is given. The point X is moving at a constant speed along the square $ABCD$ in the direction from A to B. The point Y is moving with the same constant speed along the square $BCC'B'$ in the direction from B' to C'. Initially, X and Y start out from A and B' respectively. Find the locus of all the midpoints of XY. *(Czechoslovakia)*

Second Day

4. Solve the equation
$$\cos^2 x+\cos^2 2x+\cos^2 3x=1\,.$$ *(Romania)*

5. On the circle k three points A, B, and C are given. Construct the fourth point on the circle D such that one can inscribe a circle in $ABCD$. *(Bulgaria)*

6. Let ABC be an isosceles triangle with circumradius r and inradius ρ. Prove that the distance d between the circumcenter and incenter is given by
$$d=\sqrt{r(r-2\rho)}\,.$$ *(DR Germany)*

7. Prove that a tetrahedron $SABC$ has five different spheres that touch all six lines determined by its edges if and only if it is regular. *(Soviet Union)*

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