2-nd International Mathematical Olympiad

Bucharest – Sinaia, Romania, July 18–25, 1960

First Day

1. Find all the three-digit numbers for which one obtains, when dividing the number by 11, the sum of the squares of the digits of the initial number.
(Bulgaria)

2. For which real numbers x does the following inequality hold:
$$\frac{4x^2}{(1-\sqrt{1+2x})^2} < 2x+9\ ?$$
(Hungary)

3. A right-angled triangle ABC is given for which the hypotenuse BC has length a and is divided into n equal segments, where n is odd. Let α be the angle with which the point A sees the segment containing the middle of the hypotenuse. If h denotes the height of the triangle, prove that
$$\tan\alpha = \frac{4nh}{(n^2-1)a}\,.$$
(Romania)

Second Day

4. Construct a triangle ABC whose lengths of heights h_a and h_b (from A and B, respectively) and length of median m_a (from A) are given.
(Hungary)

5. A cube $ABCDA'B'C'D'$ is given.
   (a) Find the locus of all midpoints of segments XY, where X is any point on segment AC and Y any point on segment $B'D'$.
   (b) Find the locus of all points Z on segments XY such that $\overrightarrow{ZY} = 2\overrightarrow{XZ}$.
(Czechoslovakia)

6. An isosceles trapezoid with bases a and b and height h is given.
   (a) On the line of symmetry construct the point P such that both (nonbase) sides are seen from P with an angle of 90°.
   (b) Find the distance of P from one of the bases of the trapezoid.
   (c) Under what conditions for a, b, and h can the trapezoid be constructed (analyze all possible cases)?
(Bulgaria)

7. A regular cone is given and inscribed in a sphere. Around the sphere a cylinder is circumscribed so that its base is in the same plane as the base of the cone. Let V_1 be the volume of the cone and V_2 that of the cylinder.
   (a) Prove that $V_1 = V_2$ is impossible.
   (b) Find the smallest k for which $V_1 = kV_2$, and in this case construct the angle at the vertex of the cone.
(DR Germany)

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