Kürschák Mathematical Competition 1981

1. The points of space are colored with five colors, all colors being used. Prove that some plane contains four points of different colors.

2. Let $n > 1$ be an odd integer. Prove that the necessary and sufficient condition for the existence of positive integers x and y satisfying

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y}$$

is that n has a prime divisor of the form $4k-1$.

3. In a certain country there are two tennis clubs consisting of 1000 and 1001 members. All the members have different playing skills whose orders within their clubs are known. Describe a procedure to determine in 11 games who is on the 1001st place among the 2001 players in both clubs. It is assumed that a stronger player always beats a weaker one.

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