Eötvös Mathematical Competition 1911

1. Show that, if the real numbers a, b, c, A, B, C satisfy

$$aC - 2bB + cA = 0 \quad \text{and} \quad ac - b^2 > 0,$$

then $AC - B^2 < 0$.

2. Let Q be any point on the circumcircle of a regular octagon $P_1P_2P_3 \cdots P_8$. Prove that the sum of the fourth powers of the distances from Q to the diameters P_1P_5, P_2P_6, P_3P_7, P_4P_8 is independent of the position of Q.

3. Prove that $3^n + 1$ is not divisible by 2^n for any integer $n > 1$.

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